

Combinatorics 3

17 July 2023 22:59

Problem 2.4 Compute the cardinality of the following sets.

- (i) Two digit positive odd integers.
 - (ii) Subsets of $S = \{a, b, c, d\}$ with cardinality 2.
 - (iii) Prime numbers whose base-ten digits sum to ten.
- Be careful, some have three digits

(i) $\underline{\quad} \underline{\quad} \rightarrow 1, 3, 5, 7, 9$
 $\downarrow \quad \searrow$
 $9 \quad 5$
 Total number of pos. odd integers = 45

(ii) $|S| = 4 \rightarrow |B| = 2$
 $6 = {}^4C_2 \leftarrow \{a, b\}, \{a, c\}, \{a, d\},$
 $\{ \dots \dots \dots \}$

(iii) $\underline{\quad} \underline{\quad}$

9	1	x
7	3	✓
5	5	x
3	7	✓
1	9	✓

<u>x</u>	<u>7</u>	<u>z</u>	
			1
			$x+y=9$
1	8	1	
2	7	1	
3	6	1	
4	5	1	
5	4	1	
6	3	1	
7	2	1	
8	1	1	
9	0	1	

Prove that

$$A \Delta (B \Delta C) = (A \Delta B) \Delta C$$

Ans:- $A \Delta (B \Delta C) = A \Delta ((B \Delta C) \cup (C \Delta B))$
 $= A \Delta ((B \cap C^c) \cup (C \cap B^c))$
 $= (A \cap ((B \cap C^c) \cup (C \cap B^c))^c) \cup (A^c \cap ((B \cap C^c) \cup (C \cap B^c)))$
 $= (A \cap ((B \cap C^c)^c \cap (C \cap B^c)^c)) \cup (A^c \cap ((B \cap C^c) \cup (C \cap B^c)))$

$(A \Delta B) \Delta C = (A \cap B^c) \cup (A^c \cap B) \Delta C$
 $= C \Delta ((A \cap B^c) \cup (A^c \cap B))$
 $= (C \cap ((A \cap B^c)^c \cap (A^c \cap B)^c)) \cup (C^c \cap ((A \cap B^c) \cup (A^c \cap B)))$

Venn Diagram

$$(B \cap C^c)^c = B^c \cup C$$

$$A \cap (B^c \cup C) = (A \cap B^c) \cup (A \cap C)$$

$$= B^c \cup A$$

$$C \cap (B^c \cup A)$$

$S_1 = \{ 2^n : n \in \{1, 2, \dots, N\} \}$ in terms of N

$S_2 = \{ 2^{2^n} : n \in \{1, 2, \dots, N\} \}$

$|S_1 \cap S_2|_{\text{max}} = ?$

$|S_1 \cap S_2|_{\text{min}} = ?$

... s1 ... are ?

$$|S_1 \cap S_2|_{\max} = ?$$

$$1 \rightarrow 1 \dots 2 \rightarrow \min :$$

$$S_1 \cap S_2 = \left\{ 2^n = 2^{2^k} : n \in \{1, 2, \dots, N\}, k \in \{1, 2, \dots, N\} \right\}$$

$$= \left\{ n = 2^k : n \in \{1, 2, \dots, N\}, k \in \{1, \dots, N\} \right\}$$

For $N=1$, k has to be 0 and $n=1$ not possible

$$|S_1 \cap S_2| = 0, |S_1 \cap S_2|_{\min} = 0$$

$$n < 2^n \text{ for } n \geq 1$$

$$|S_1 \cap S_2|_{\max} = \log_2 N$$

for $k < n$ we can have $2^k = n$

$$N=4, \begin{matrix} n=2, k=1 \\ n=4, k=2 \end{matrix} \Rightarrow \log_2 4 = \log_2 N$$

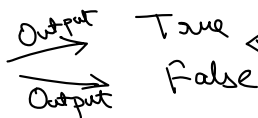
$$\boxed{\begin{matrix} a^b = c \\ b = \log_a c \end{matrix}}$$

Q:- Prime numbers whose base-10 digits sum to 10.

Ans:- $\text{dig-sum}(19) = 1+9 = 10$

\Rightarrow Infinite (later)

Q:- $A \cup B = A \cap C$



$\{x : \underbrace{A \cup B = A \cap C}_{\text{Conditional statement with boolean output}}\}$

Boolean Operation & Operator :-

Operator is •

Let a, b be two variables then, $a \bullet b$ is another variable with a boolean operator and a boolean operation is done among a and b

Like $(+)$ is a boolean operator

$(-), (=), (x)$ " "

$a \bullet b$ is the boolean operation

Like $(+)$ is a boolean operation
 $(-)$, (\div) , (\times) " " "

A^c is not a boolean operation as ' c ' is done on one variable
→ This is an unary operation

$F = \{x_1, x_2, x_3, x_4, \dots\} \rightarrow$ Fibonacci

$x_1 = 0, x_2 = 1$ (generally)

$x_3 = x_2 + x_1, x_4 = x_3 + x_2, x_n = x_{n-1} + x_{n-2}$

Q:- Suppose we have a set S with $n \geq 0$ elements. Find the formula for the number of different subsets of S that has k elements

Ans:- $nC_k =$ as we will be choosing only k elements from n elements
(no ordering needed as it is a subset)

$n=3, k=2$
 $\{a, b, c\}$

$\{a, b\}, \{b, c\}, \{c, a\}$
